May 19, 2019

Professor Martin Kubala
Dean of the Faculty of Science
Palacký University in Olomouc

Dear Professor Kubala,

I write to share with you my assessment of Jan Kühr's record, who is under consideration for promotion to full professor. I have known Jan for about fifteen years. He has visited Vanderbilt for a number of times, I have visited Palacký University once, we have met in numerous disciplinary conferences, and we have been research collaborators in two research articles (one published and the other in the final stages of preparation). Thus, I can speak with confidence and objectivity about his scholarship and the major place he has carved out for himself in algebraic logic and algebra. He is one of the most creative and productive scholars presently engaged in these fields, and the continuous rise of his reputation is the result of exceptional intellectual capacity, strong work ethic and good taste for mathematics.

A matter of primary importance in promotion cases to the rank of full professor is the number and quality of scholarly articles published by the candidate. The candidate's record must leave no doubt about his leadership as a research scholar and the national and international visibility he has attained. In particular, it must include published or accepted work in leading field journals. Jan's research record shines when it is compared with that of a very elite group of peers. In my capacities as Chair, Vice-Provost and Executive Dean, I have had the opportunity to conduct or evaluate more than two hundred faculty promotion, appointment and reappointment cases representing all disciplines in the College of Arts and Science at Vanderbilt University. In addition, I have been asked over the years to evaluate the research records of numerous colleagues in the U.S. and abroad. Hence, I feel confident to state that Jan is a rare scholar, and one of the strongest among a large group of mid-career researchers throughout the world who share research interests with him.

Jan's list of publications is impressive and consists of two books and fifty-four published articles. Moreover, the list of journals in which his papers have appeared includes such prestigious journals as Journal of Algebra, Algebra Universalis, Order, Fuzzy Sets and Systems, Soft Computing, Mathematica Slovaca, Czechoslovak Mathematics Journal and International Journal of Theoretical Physics. His independence as a research scholar is also evident. While a lot of his work is collaborative, he has seventeen high quality single-authored articles. Also, while it would be inappropriate for me to evaluate the strength of our two joint papers (one in preparation), I am comfortable to state that he has been an equal partner in our collaboration.

There are other indicators of his international visibility, such as the significant number of talks in international conferences, visits in various institutions, editorial work, etc.

A large part of Jan's work involves algebras, as well as their reducts, arising as algebraic counterparts (semantics) of non-classical logics. In particular, a significant part of his research involves classes of residuated lattices, which may be viewed as the algebraic counterparts of (propositional) substructural logics. Substructural logics are non-classical logics that are weaker than classical logic, in the sense that they may lack one or more of the structural rules of contraction, weakening and exchange in their Genzen-style axiomatization. These logics encompass a large number of non-classical logics related to computer science (linear logic), linguistics (Lambek Calculus), philosophy (relevant logics), and many-valued reasoning. Residuated lattices first appeared explicitly in the work of Krull, Ward and Dilworth as abstractions of ideal lattices of rings in the early 1930's. Their study, however, goes even further back to Hilbert's foundational studies of geometry, and Riesz's development of the theory of operators and their spaces. Thus, residuated structures have played an important role in mathematics independently of their connection with substructural logics. The next couple of paragraphs mentions just a few of the highlights of Jan's work, which will hopefully demonstrate his versatility and depth as a top research mathematician. (The numbers [.] refer to the list of papers in his curriculum vitae.)

Jan obtained significant results from the very beginning of his research career. In his 2003 Ph.D. thesis, he studied the class of DRℓ-monoids. Around the same time, my collaborators and I studied independently the class of GBL-algebras, which is equivalent to the aforementioned class. His objective, following a suggestion by Petr Hájek, was to define a semilinear, non-commutative version of BL-algebras. He did that successfully and provided a far reaching extension of Paul Conrad's results for lattice ordered groups to the setting of residuated lattices. In particular, he made an extensive study of the lattices of convex subalgebras and normal convex subalgebras (he used the terms "ideals" and "normal ideals", respectively) for these algebras, and axiomatized semilinear ("representable") pseudo-BL-algebras in [52], an article that drew considerable attention, and later semilinear DRℓ-monoids in [53]. Further, he studied values (normal, special, essential) and complete distributivity in the case of DRℓ-monoids that are equivalent to integral GMV-algebras. He also employed Anatolij Dvurečenskij's generalized pseudo-effect algebras to prove that the lattice of convex subalgebras ("ideals" in his terminology) of such an algebra is isomorphic to the lattice of convex ℓ-subgroups of a suitable ℓ-group.
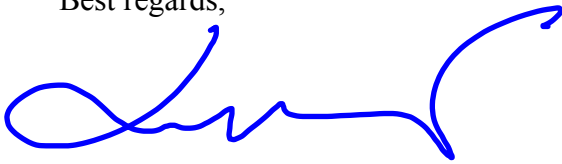
He subsequently directed his attention to pseudo-BCK-algebras, which was the main topic of his 2007 habilitation thesis. This work included a generalization of Ono-Komori's construction that led to the proof of the interesting result that every pseudo-BCK-algebras are the residuation subreducts of integral residuated lattices. Perhaps the strongest result in this setting, published in [35], was a proof of Clint van Alten's conjecture that provided an axiomatization of semilinear pseudo-BCK-algebras (van Alten used the term "biresiduation algebras"). Furthermore, he proved two variants of Cantor-Bernstein theorem for orthogonally sigma-complete pseudo-BCK-

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algebras [29], [21]. In a joint project with Dvurečenskij [22], he characterized linearly ordered pseudo-BCK-algebras that are ordinal sums of Bosbach's linearly ordered cone algebras.

Last, but certainly not least, he has produced a substantial body of work with Ivan Chajda and Radomír Halaš. Two references of note are their book on semilattice-ordered structures in 2007 and their introduction of the concept of "basic algebra" in [25]. A byproduct of their work is the fact that lattice-ordered effect algebras form a variety, which led to a number of interested results about varieties of these algebras, e.g. [11] and [6]. For example, [6] provides a simple axiomatization of the varietal join of MV-algebras and orthomodular lattices as subvarieties of lattice-ordered effect algebras. (Incidentally, he has more unpublished results along these lines for varieties generated by horizontal sums of MV-algebras.)

These are just few examples of Jan's many and multifaceted research contributions. He is approaching the most productive and impactful years of his professional life. I have no doubt that he will continue producing field defining and ground-breaking contributions for many years to come. I recommend him to you for promotion to the rank of full professor without any reservation and with great enthusiasm.

Best regards,

Constantine Tsinakis
Professor of Mathematics